Mathematica 11.3 Integration Test Results

Test results for the 14 problems in "Bronstein Problems.m"

Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 252 leaves, 3 steps):

$$\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}+$$

$$\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

Result (type 4, 122 leaves):

$$\frac{2\,(-1)^{1/6}\sqrt{(-1)^{5/6}\,(-1+x)}\,\sqrt{1+x+x^2}\,\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)}{3^{1/4}\sqrt{1-x^3}}$$

Problem 6: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{-71-96x+10x^2+x^4}}\,dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\frac{1}{8}\,\text{Log}\left[10001+3124x^2-1408x^3+54x^4-128x^5+20x^6+x^8+\sqrt{-71-96x+10x^2+x^4}\,\left(-781+528x-27x^2+80x^3-15x^4-x^6\right)\right]$$

Result (type 4, 1226 leaves) :

$$-\Bigg(\Bigg(2\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\Bigg(\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}\right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]\right)}{\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]\right)}}\right],\right.$$

$$\left.\frac{\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,3\right]\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]\right)}{\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,3\right]\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]\right)}\right]-$$

$$4\sqrt{2\left(-1+\sqrt{3}\right)}\;\text{EllipticPi}\left[\frac{\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]}{\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]},\right.$$

$$\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]\right)}{\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]\right)}}\right],$$

$$\left.\frac{\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,3\right]\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]\right)}{\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,3\right]\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\sharp 1+10\,\sharp 1^2+\sharp 1^4\,\&,\,4\right]\right)}\right]\Bigg)$$

$$\sqrt{\frac{x-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,3\right]}{\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,3\right]\right)}}$$

$$\sqrt{\frac{\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,4\right]\right)}{\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,4\right]\right)}}$$

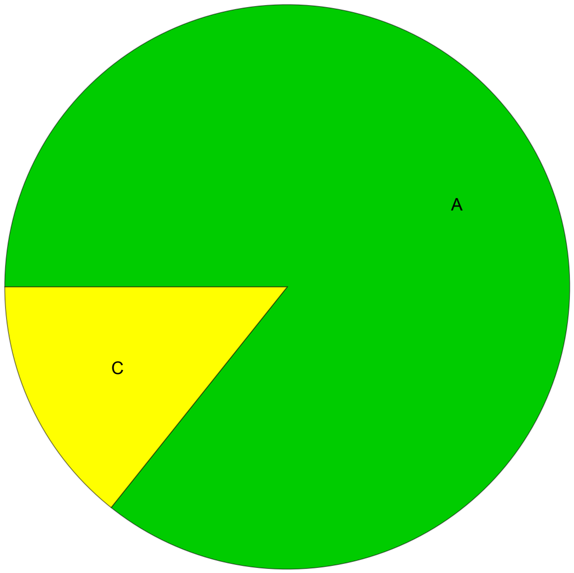
$$\left.\left(x-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,4\right]\right)\right)\Bigg/$$

$$\left(\sqrt{-71-96\,x+10\,x^2+x^4}\;\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,4\right]\right)\right.$$

$$\left.\left.\sqrt{\frac{x-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,4\right]}{\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,4\right]\right)}}\right)\right)$$

Summary of Integration Test Results

14 integration problems



A - 12 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 2 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts